Trigonometric altitude determination

Patrick Reimann[1]

[1] Amt für Geoinformation BL, Liestal, Switzerland

patrick.reimann@bl.ch

Part of this programme: swisstopo, ETH Zürich

Keywords: altitude calculation and error theory; long distances; refraction; curvature of the earth; gravitational field and -anomaly; context to altitude determination Mount Everest

1. Introduction

The rapport from 2019 covered the theory of trigonometric altitude determination in detail – as well as the fact that George Everest in 1852 determined the altitude of Mount Everest, later named after him, from a distance of 200 km as 8840 m - the value valid today is 8848 m!

According to several press reports in Nov 20, China and Nepal started a project between 2018 and 2019 to re-determine the altitude of Mt. Everest. Both teams were on the summit independently and measured elevation angles to surrounding points known by GPS coordinates. In order to do justice to the geoid (see chapter 2), gravity measurements were also taken there. The new altitude of Mt. Everest thus determined by mutual agreement is 8848.86 m above sea level. With this new scientific result, admiration for the 19th century work of George Everest increases.

George Everest may have read off the decimal places of the elevation angles in a vernier scale, as is necessary for this research work on the Kern repetition theodolite, which was involved in 1909.

Figure 1. Altitude determination Mönch with Kern 1909.

The reading of the elevation angle is done laterally with the additional difficulty of reading the second and third decimal places with a 20-nonius (Figures 2 and 3).

Figure 2. Altitude reading analogous to the procedure of George Everest.

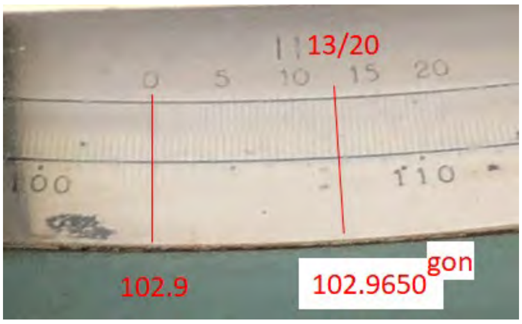

Figure 3. Altitude reading 102.9 + 13/20 = 102.9650 gon.

The vernier scale was introduced in 1631 by the French mathematician Pierre Vernier (1580-1637) and used in historical measuring instruments. With it, the angle sought could be indicated more precisely than the human eye could recognize.

2. Systematic deviations

Geoid and gravity field

The geoid is a selected equipotential surface of the Earth's gravitational field and serves as a reference surface for altitude determination. It can be thought of as an idealised mean sea surface that continues under the continents [1].

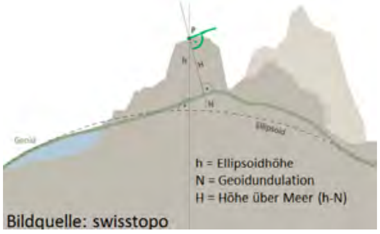

Figure 4. Influence of the geoid on the horizon of the instruments and on the effective altitude above sea level; not shown to scale.

The current geoid model of Switzerland (CHGeo2004) was determined from a combination of astro-geodetic and gravimetric methods and has an accuracy of 1-3 cm over the whole of Switzerland.

The difference to the ellipsoid is called geoidundulation. The lowest value in Switzerland is -4 m in the southern Ticino, the highest value + 4 m in the Engadine.

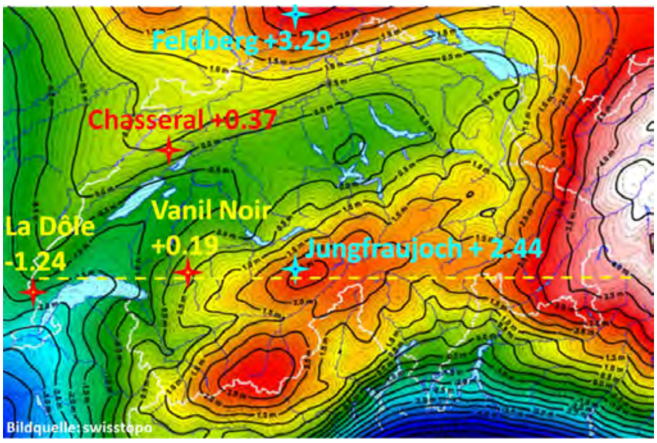

Figure 5. Swiss geoid model (CHGeo2004).

However, the look through the telescope does not take into account the course of the geoidundulation. Thus, the altitude calculation for La Dôle, for example, must be reduced by 3.68 m (Figures 9 and 10).

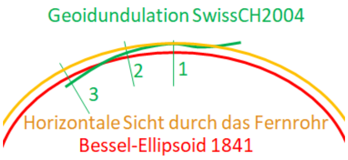

Figure 6. Geoidundulation 1 Jungfraujoch (+2.44), 2 Vanil Noir (+0.19), 3 La Dôle (-1.24).

The connection between the geoidundulation and the perpendicular deviation is not yet clarified, namely whether further systematic reductions have to be applied to the calculated altitude for this reason.

Temperature and refraction

As already shown, the refraction is dependent on the temperature. At low temperatures, a constant refraction value (0.13) should result in higher target altitudes than at higher temperatures. In other words, with two measurements and identical good visibility conditions but different temperatures, the observer receives differently calculated altitudes.

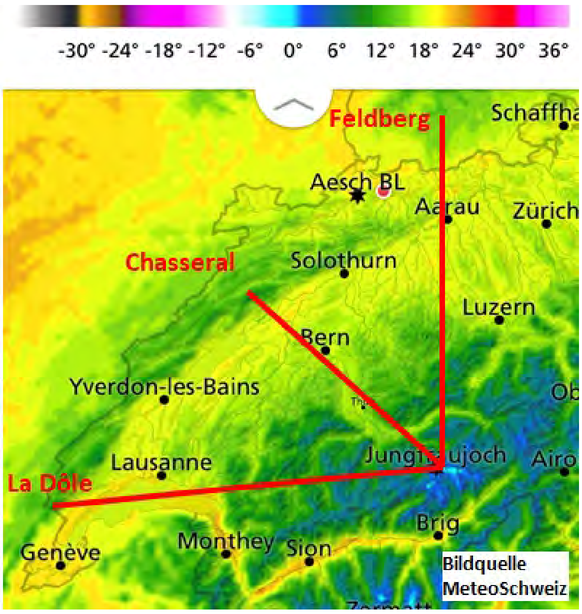

Figure 7. Temperature animation on 26 May 2020 at 1pm.

Whether and how the change in elevation angle is related to the temperature gradient along the sighting will have to be investigated in another context.

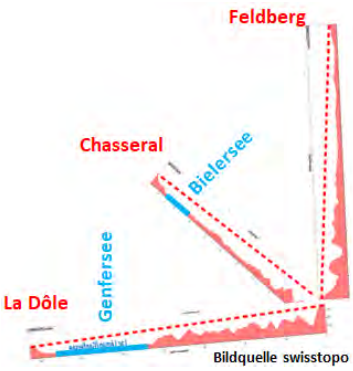

Figure 8. Longitudinal profiles of remote destinations.

Also of interest is whether the size of the clear height below the views plays a role in the refraction and whether lake surfaces reduce or increase the sensitivity of the refraction.

[1] https://www.swisstopo.admin.ch/ ⇨ knowledge and facts ⇨Surveying / Geodesy ⇨ Geoid

3. Measurement program 2020

The 2020 measurements focused primarily on points known by coordinates at far, medium and near distances (Figures 4-6).

The majority of these are triangulation points with a known altitude – or buildings, masts or other features that are directly next to them and are also clearly visible from a distance.

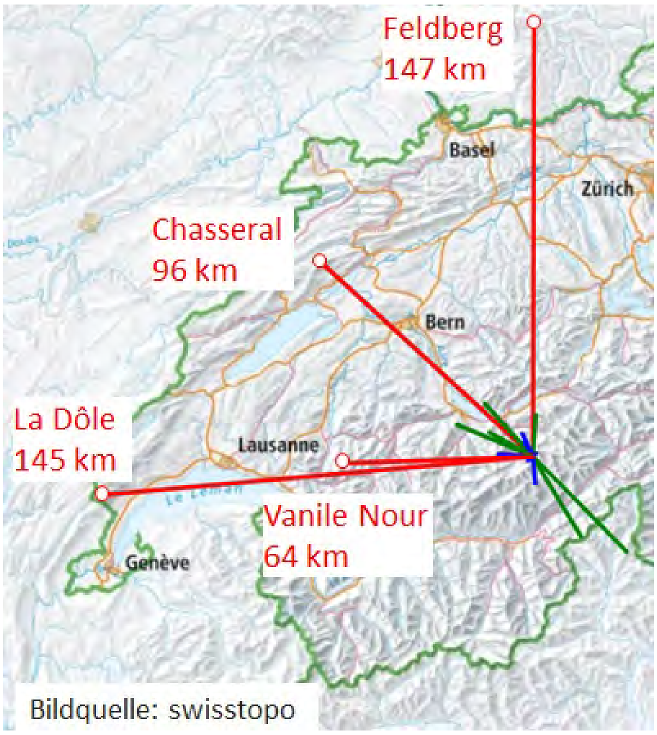

Figure 9. Far distances.

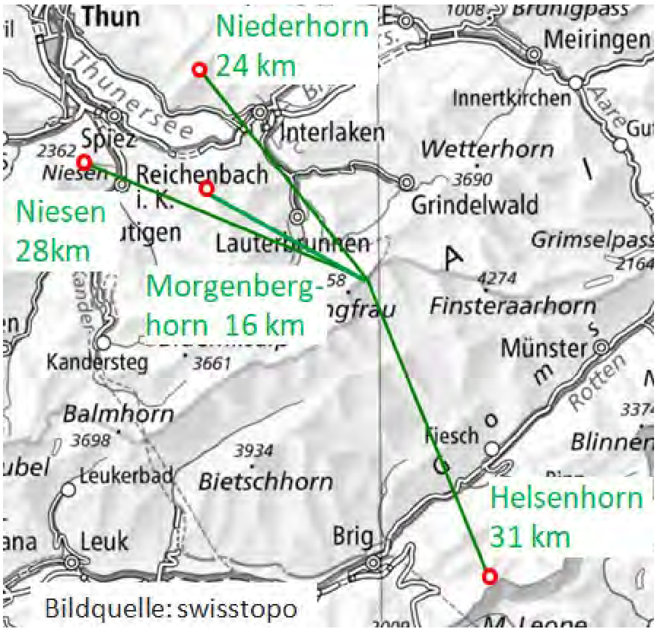

Figure 10. Medium distances.

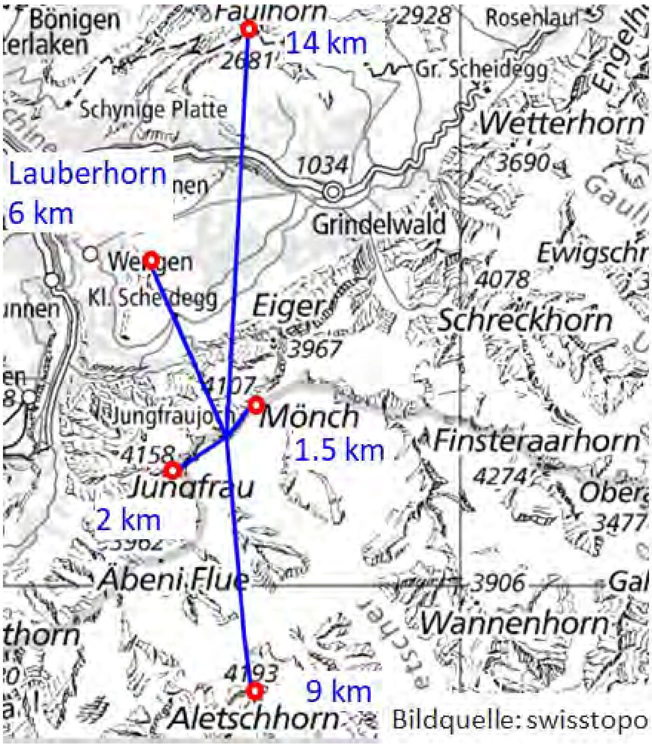

Figure 11. Near distances.

For the Helsenhorn, Mönch, Jungfrau and Aletschhorn, the measured altitudes are compared with the values from the national map.

To confirm the systematic dependence of the refraction on the temperature, the temperature at the site is also recorded for each measurement in addition to the elevation angle.

4. Results 2020

The evaluations of the selected target locations uniformly concern the results from the measurements with the most advanced instrument Leica 1100. However, they are confirmed in a first approximation with samples with the older instruments Wild T2 (1952) and Kern (1909).

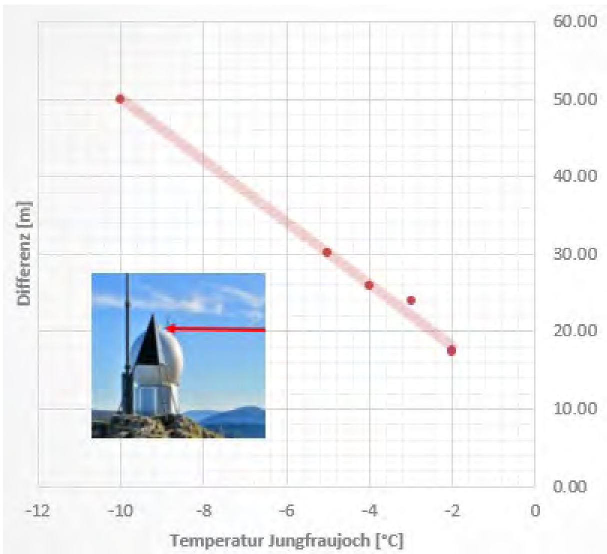

Figure 12. La Dôle, average difference 27 m.

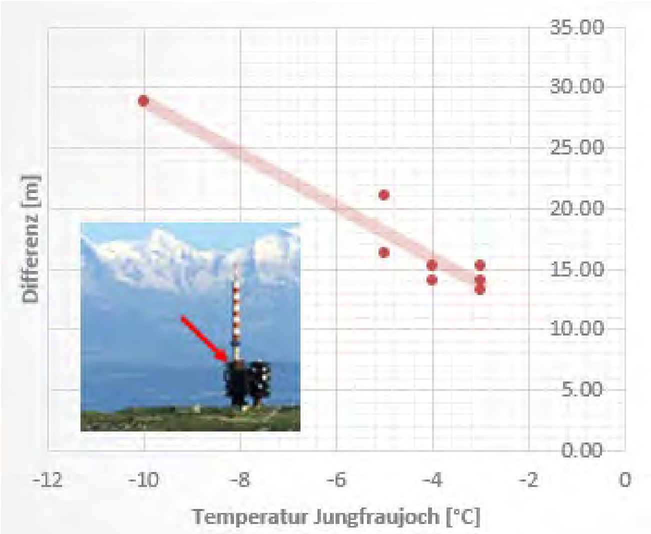

Figure 13. Chasseral, average difference 15 m.

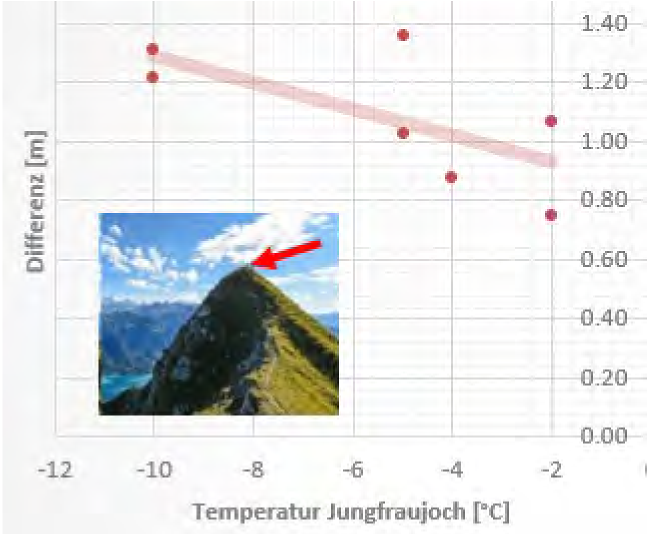

Figure 16. Morgenberghorn, average difference 1.1 m.

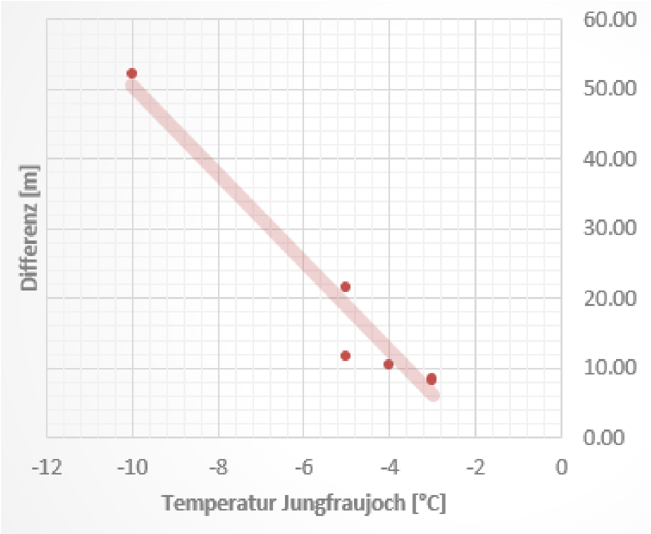

Figure 14. Feldberg, average difference 21 m.

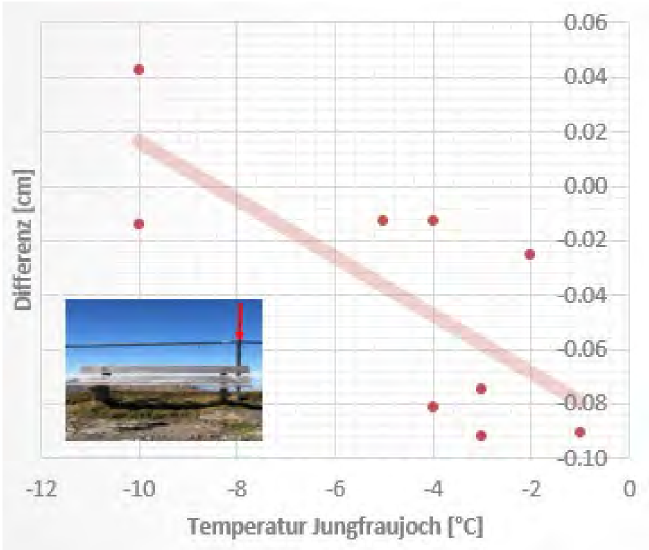

Figure 17. Lauberhorn, average difference -4cm.

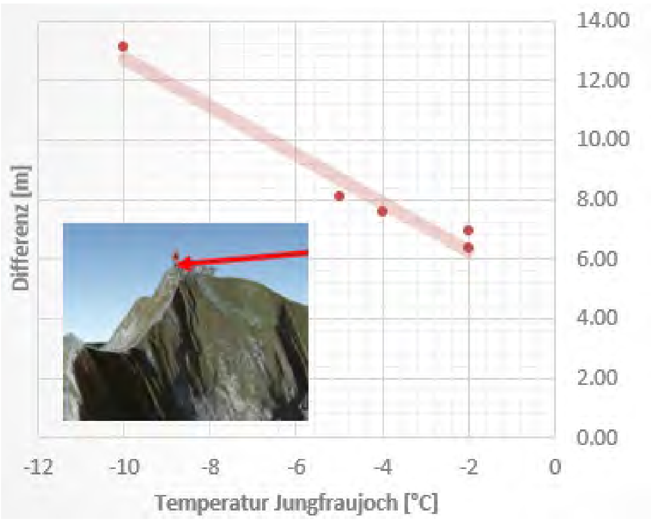

Figure 15. Vanil Noir, average difference 8 m.

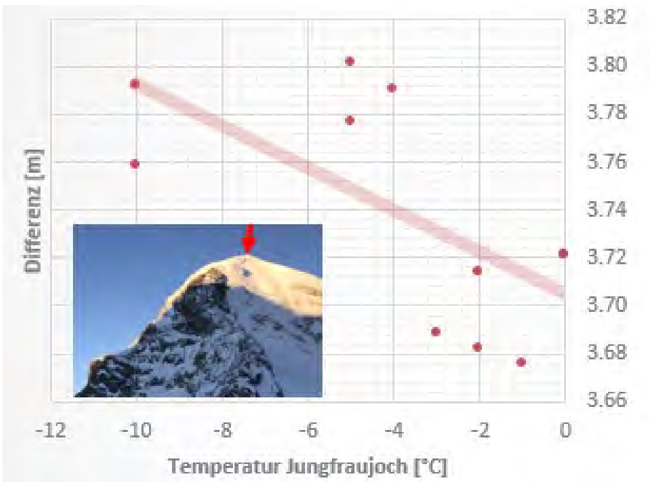

Figure 18. Mönch, average difference 3.7m.

5. Assessment

According to these calculations, the comprehensive measurements from 2020 confirm a linear dependence between the refraction with the temperature at the site as told the theory.

The variance of the individual evaluations results from different visibilities and possibly also from the observer's form of day.

However, it may be noted that these results may be considered representative for the study of refraction.

The averaged values of the differences are considerable at the current stage of this project, especially when considering the longest view to La Dôle (145 km, 27m). This is also due to the fact that the exact altitude of the target is known.

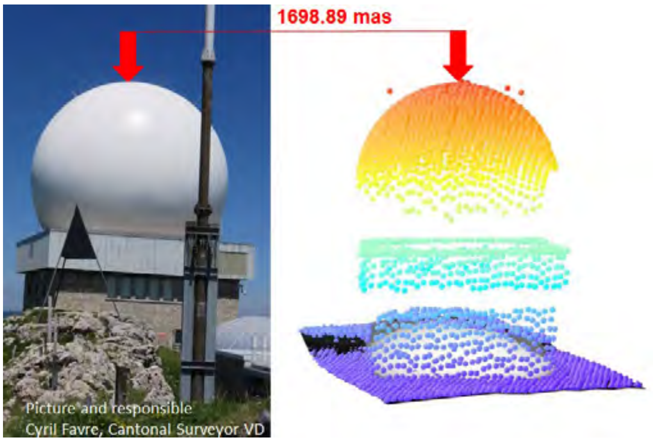

Figure 19. Absolute altitude of La Dôle building from LiDAR data.

The small average difference to the Lauberhorn (6 km, -0.04m) is interesting, although it is close enough to expect such good results.

Even closer than Lauberhorn is Mönch, a four-thousand-metre peak in the Bernese Alps. The average difference of 3.70 metres compared to the national map draws attention. The observer represents his repeatedly determined altitude and swisstopo has already conceded. There will be something to report next year.

Overall, the mean differences cannot yet be summarised in a uniform error theory. The measures in chapter 6 could make this possible.

6. Outlook

Stephen W. Hawking wrote in his book "A Brief History of Time": 'A theory is good if it fulfils two conditions: It must contain **a large number of observations** and thus describe the basis of a model that contains **only a few arbitrary elements**'.

The results from chapter 4 are remarkable, but for a scientific evaluation, a much higher number of measurements under meteorologically clear conditions is necessary. Regarding refraction, studies at ETH Zurich have shown that the absolute temperature at the site does allow statements to be made about the effect of altitude determination. The investigation becomes more accurate if the temperature gradients are recorded over the first 3 meters from the viewpoint.

Finally, the possibility of systematic perpendicular deviation of the site should be investigated. The ETH Zurich and the Federal Office of Topography swisstopo have documents on the Jungfraujoch that should be included in the study.

Furthermore, the results of the altitude difference are of interest if measurements are taken simultaneously on the Jungfraujoch and at one of the target locations. Simultaneous reciprocal measurements mathematically and thus theoretically resolve the influence of refraction.

To be continued.

References

Eschmann 'Ergebnisse der trigonometrischen Vermessungen in der Schweiz'; 1840.

Engi 'Zur trigonometrischen Höhenmessung im Gebirge'; 1951.

Chaperon/Elmiger 'Vermessungskunde'; 1986.

Stephan W. Hawking 'Eine kurze Geschichte der Zeit', ISBN 3 499 18850 3.

Brocks 'Vertikaler Temperaturgradiant und terrestrische Refraktion insbesondere im Gebirge'; 1939.

Wolfgang Torge Geodesy, Walter de Gruyter 1991.

Gerber MAS 'Untersuchung von Refraktionseffekten in alpinem geodätischem Monitoring'

Internet data bases

https://www.swisstopo.admin.ch/ ➪ knowledge and facts ➪ Surveying / Geodesy ➪ FAQ ➪ terrestrial curvature

Collaborating partners / networks

Prof. Dr. Andreas Wieser; Departement Bau, Umwelt und Geomatik, Institut für Geodäsie und Photogrammetrie, ETH Zürich
Dr. Urs Marti, Bundesamt für Landestopografie swisstopo
Julia Wunsch, Assistenz und Photographie
Cyril Favre, Kantonsgeometer VD
Johannes Gerber, Flotron AG, 3860 Meiringen
Ruedi Käser, Vermessungsingenieur HTL und Betriebsleiter HFSJG
Karl Heinz Münch, Studiensammlung Kern AG

Address

Amt für Geoinformation BL
Mühlemattstrasse 36
CH-4410 Liestal
Switzerland

Contacts

Patrick Reimann, Eidg. Pat. Ing.-Geometer
Tel.: +41 61 552 5685
e-mail: patrick.reimann@bl.ch